Studies on the Performance of ITER90H-P Fusion Reactor Considering the D-T and D-\(^3\)He Fuel in the Steady-state

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**ABSTRACT**

In this work, the performance of fusion reactor ITER90H-P with considering D-T and D-\(^3\)He fuels areexamined by writing the dynamics equations on thesystemreactor. Therefore, we solve these equations analytically in thesteady state. In this state we determine the optimum conditions for achieving the maximum fusion gain. In addition, we ignore the impurities because we need to high performance points without impurities. Our calculationsinthis papershow that we have maximum fusion gain for D-T and D-\(^3\)He fusion reactions in steady state at resonance temperature Kev70 for D-T fusion reaction respectively .Their maximum values of fusion gain  are equal to 6.01 for D-T fuel-and the 0.012 for D-\(^3\)He, respectively. Therefore, currently, using D-\(^3\)He as a fusion fuel is not recommended.

Key words: steady, fusion gain, power, helium, tritium


**INTRODUCTION**

The aim of fusion research is to utilize the energy source of the sun and stars here on earth: A fusion power plant is to derive energy from fusion of atomic nuclei. Under terrestrial conditions this can most readily be achieved with the two hydrogen isotopes, deuterium and tritium. These fuse to form helium, thus releasing neutrons and large quantities of energy: One
game of fuel could yield in a power plant 90,000 kilowatt-hours of energy, i.e. the combustion heat derived from 11 tons of coal. The basic substances needed for the fusion process, viz. deuterium and lithium, from which tritium is produced in the power plant, are available throughout the world in almost inexhaustible quantities. Like a coal fire, a fusion fire does not happen on its own, but only when the appropriate ignition conditions are present. As regards the fuel— a low-density, ionized gas, a “plasma” it needs an ignition temperature of 100 million degrees. This high temperature precludes the plasma from being directly confined in material vessels. Any wall contact occurring would immediately cool the hot gas. Instead, use is made of magnetic fields, which confine the fuel as thermal insulation and keep it away from the vessel walls. The principle of deriving energy in this way was first realized in the JET (Joint European Torus) device at Culham, UK, the world’s largest fusion experiment at present. It was jointly planned and built by Europe’s fusion scientists and has also been jointly operated since 1983. All scientific and technical objectives specified in the planning have meanwhile been attained or even exceeded. In 1997 a transient fusion power of 16 megawatts was achieved. More than half the power needed to heat the plasma was regained through fusion. However, the JET plasma with its volume of 80 cubic meters is too small to provide a net energy gain. This will be the role of the ITER (Latin for the “way”) international experimental reactor. In its plasma volume of about 830 cubic metres a fusion power of 500 megawatts is to be produced, this being ten times as much as is needed to heat the plasma. Human increasing need for energy has led to significant progresses to be done in achieving the values of temperature, density and confinement time and required parameters to build a fusion power plant. One of the major projects in which scientists, designers and engineers from different countries are doing, to design a nuclear fusion reactor, is ITERtokomak. The ITER project itself began at the Geneva Summit in 1985, with a device designed to be capable of a steady-state, self-sustaining fusion reaction with a significant net energy gain. ITER Conceptual Design Activities (CDA) began in 1988 and were completed in 1990, carried out jointly by the U.S., E.U., Japan and Russia under the auspices of the IAEA. Engineering Design Activities (EDA) commenced in 1992 and finished in 1998 resulting in a complete design. Financial constraints demanded a reduced-cost approach, though, and a second EDA period of 1999-2001 completed the current ITER-FEAT (Fusion Energy Amplifier Tokamak) design (Figure 1).

Figure 1: The ITER-FEAT device and major components
With these improvements, it is hoped that ITER will also allow for the possibility of reaching a more important goal, one that will be essential for a fusion power plant. For a deuterium-tritium plasma, once heating of alpha particles (the Helium nuclei product of fusion), not by external input but by the fusion process itself, is equal to the heat loss through the vessel walls and diverter, the plasma becomes self-sustaining and is said to be ignited, or burning. External heating can be turned off, and the plasma will continue to exist and induce fusion. With no heating (energy input), the Q factor ratio tends to infinity and the fusion process is controlled in steady state only by the fuelling rate to the torus.

In order for nuclear fusion reactor is economically affordable; the system must stay for a long time in the burning plasma steady-state with performance point’s at high Q. Here Q is the ratio of auxiliary power to fusion power which is called fusion. Thus, in this paper, underthese conditions we will have study on the behavior of ITER 90 HP fusion reactor in the steady-state for two fusion fuel such as D-T and D-3He with taking into account the system temperature variations. Finally, we determine the required conditions to achieve high fusion gain. The rest of the paper is organized as follows: in section 2, Reactivity parameter for D-T and D-3He fusion reactions is presented. In section 3, Nonlinear point Kinetic equations governing on the ITER 90 HP fusion reactor ITER 90 HP for the D-T and D-3He fuel are stated, and these equations are solved analytically. Section 4 is concerned with these equations are solved analytically. Section 5 discusses the results of the Calculation and comparison of required parameters for study of ITER 90 HP fusion reactor at steady state for both fuel D-T and D-3He. Finally, section 6 contains some conclusions concerning this work and further extensions and research suggested in this direction.

**Table 1: ITER machine parameters**

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>QUANTITY</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Plasma Current</td>
<td>22.0MA</td>
</tr>
<tr>
<td>R</td>
<td>Minor Radius</td>
<td>2.15m</td>
</tr>
<tr>
<td>A</td>
<td>Major Radius</td>
<td>6.0m</td>
</tr>
<tr>
<td>B</td>
<td>Magnetic Field</td>
<td>4.85T</td>
</tr>
<tr>
<td>k_x</td>
<td>Elongation at X</td>
<td>2.2</td>
</tr>
<tr>
<td>k_a</td>
<td>Alpha particle confinement Cte</td>
<td>7</td>
</tr>
<tr>
<td>k_DT</td>
<td>DT particle confinement Cte</td>
<td>3</td>
</tr>
<tr>
<td>k_l</td>
<td>Impurity particle confinement Cte</td>
<td>10</td>
</tr>
<tr>
<td>β_max</td>
<td>Beta limit</td>
<td>2.51(1)B=5.3%</td>
</tr>
<tr>
<td>V</td>
<td>Plasma volume</td>
<td>1100m³</td>
</tr>
</tbody>
</table>

**Reactivity parameter for D-T and D-3He Fusion Reactions**

This paper presents a strategy for the development of D-3He fusion for terrestrial and space power. The approach relies on modest plasma confinement progress in alternate fusion concepts and on the relatively less challenging engineering, environmental and safety features of a D-3He fueled fusion reactor compared to a D-T fueled fusion reactor. The D-3He benefits include full-lifetime materials reduced radiation damage,
less activation, absence of tritium breeding blankets, highly efficient direct energy conversion, easier maintenance and proliferation resistance. The main fusion fuels are:

\[ \text{D + T} \rightarrow ^4\text{He} + n + 17.6 \text{ MeV} \]
\[ \text{D + } ^3\text{He} \rightarrow ^4\text{He} + p + 18.4 \text{ MeV} \]

Also another important parameter is reactivity of D - T and D - ^3He which depends on the temperature.

a) Bucky Reactivity is temperature dependent \((T \text{ (keV)})\) and is given by:

\[ \langle \sigma v \rangle_{\text{DT}} = \exp \left( \frac{a_1}{T^r} + a_2 + a_3T + a_4T^2 + a_5T^3 + a_6T^4 \right) \]  

(1)

Here \(a_i\) and \(r\) are given in the Table 2.

Table 2: Numerical values of \(a_i\) and \(r\) parameters for D - T and D - ^3He fusion reactions using Bucky formula.

<table>
<thead>
<tr>
<th></th>
<th>D-3He</th>
<th>DT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>2.1377692×10^1</td>
<td>2.7764468×10^1</td>
</tr>
<tr>
<td>(a_2)</td>
<td>2.5204050×10^1</td>
<td>3.1023898×10^1</td>
</tr>
<tr>
<td>(a_3)</td>
<td>1.1703427×10^-2</td>
<td>2.7889999×10^-2</td>
</tr>
<tr>
<td>(a_4)</td>
<td>1.937545×10^-4</td>
<td>5.5321633×10^-4</td>
</tr>
<tr>
<td>(a_5)</td>
<td>4.9246592×10^-6</td>
<td>3.0293927×10^-6</td>
</tr>
<tr>
<td>(a_6)</td>
<td>3.9836572×10^-8</td>
<td>2.5233325×10^-8</td>
</tr>
<tr>
<td>(r)</td>
<td>0.2935</td>
<td>0.3597</td>
</tr>
</tbody>
</table>

b) Bosch-Halereactivity is given by the following formula

\[ \langle \sigma v \rangle = C_1 e^{-\xi} \sqrt{\left( m_c^2 C^2 T^3 \right)} \]  

(2)

\(\xi\), \(\theta\) and \(B_G\) are:

\[ \xi = \left( \frac{B_G^2}{4\theta} \right)^{\frac{3}{2}} \]  

(3)

\[ \theta = T \left[ 1 - \frac{T\left(C_2 + T\left(C_4 + T(C_6)\right)\right)}{1 + T\left(C_3 + T\left(C_5 + T(C_7)\right)\right)} \right] \]  

(4)

\[ B_G = \pi \alpha Z_1 Z_2 \sqrt{2m_c^2} \]  

(5)

The constants values of \(C_1\) to \(C_7\) in these equations for different fusion reactions are given in Table (3).
Table (3): The constants values of $C_1$ to $C_7$ for different fusion reactions[4]

<table>
<thead>
<tr>
<th></th>
<th>DT</th>
<th>DDN</th>
<th>DDP</th>
<th>D3He</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>1.17E-09</td>
<td>5.43E-12</td>
<td>5.66E-12</td>
<td>5.51E-10</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.51E-02</td>
<td>5.86E-03</td>
<td>3.41E-03</td>
<td>6.42E-03</td>
</tr>
<tr>
<td>$C_3$</td>
<td>7.52E-02</td>
<td>7.68E-03</td>
<td>1.99E-03</td>
<td>-2.03E-03</td>
</tr>
<tr>
<td>$C_4$</td>
<td>4.61E-03</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>-1.91E-05</td>
</tr>
<tr>
<td>$C_5$</td>
<td>1.35E-02</td>
<td>-2.96E-06</td>
<td>1.05E-05</td>
<td>1.36E-04</td>
</tr>
<tr>
<td>$C_6$</td>
<td>-1.07E-04</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>$C_7$</td>
<td>1.37E-05</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>m, e²</td>
<td>1124656</td>
<td>937814</td>
<td>937814</td>
<td>1124572</td>
</tr>
</tbody>
</table>

According to the above equations and the data in Tables (2) and (3) we plotted the $\langle \sigma v \rangle$ versus temperature for the fusion reaction of D – T and $D - ^3He$ for both of Bucky and Bosch-Hale formulae.

![Graph](image)

**Figure2:** Comparison of the graphs of fusion reactions reactivity variations for a) D - T and b) $D - ^3He$ versus temperature by the two methods, Bucky $\langle \sigma v \rangle_{DT,B} \left( cm^3.s^{-1} \right)$, $\langle \sigma v \rangle_{D^3He,B} \left( cm^3.s^{-1} \right)$ and Bosch-Hale $\langle \sigma v \rangle_{DT,B-H} \left( cm^3.s^{-1} \right)$ $\langle \sigma v \rangle_{D^3He,B-H} \left( cm^3.s^{-1} \right)$

As shown in Figure2-a, the reactivity of D – T fusion reaction is greater than $D - ^3He$. Because $< \sigma v >_{DT}$ at 70 kev temperature has a maximum value thus 70 kev is temperature resonance. the value of D – T reactivity at this temperature approximately 10 times is greater than $D - ^3He$. By viewing the obtained numerical values and Figure2-b we find that the difference between the two ways of calculating reactivity is minimal and since that the method of Bucky is newer than Bosch-Hale in our calculations we use this.
NONLINEAR POINT KINETIC EQUATIONS GOVERNING ON THE ITER 90 HP FUSION REACTOR ITER 90 HP FOR THE D - T AND D - 3He FUEL

In this work, we have used fusion reactor in which approximately particle energy balance equations for two D - T and D - 3He fuel are given by:

a) Nonlinear point kinetic equations governing on the D - T fuel

\[
\frac{dn_a}{dt} = -\frac{n_a}{\tau_a} + \left(\frac{n_{DT}}{2}\right)^2 <\sigma v>_{DT} \\
\frac{dn_{DT}}{dt} = -\frac{n_{DT}}{\tau_{DT}} - 2\left(\frac{n_{DT}}{2}\right)^2 <\sigma v>_{DT} + \frac{n_n}{\tau_d} \\
\frac{dn_n}{dt} = -\frac{n_n}{\tau_d} + S \\
\frac{dn_l}{dt} = -\frac{n_l}{\tau_I} + S_I \\
\frac{dE}{dt} = \frac{-E}{\tau_E} + P_\alpha + P_{\text{ohmic}} - P_{\text{rad}} + P_{\text{aux}}
\]

b) Nonlinear point kinetic equations governing on the D - 3He fuel

\[
\frac{dn_a}{dt} = -\frac{n_a}{\tau_a} + \left(\frac{n_{D^3He}}{2}\right)^2 <\sigma v>_{D^3He} \\
\frac{dn_{D^3He}}{dt} = -\frac{n_{D^3He}}{\tau_{D^3He}} - 2\left(\frac{n_{D^3He}}{2}\right)^2 <\sigma v>_{D^3He} + \frac{n_n}{\tau_d} \\
\frac{dn_n}{dt} = -\frac{n_n}{\tau_d} + S \\
\frac{dn_l}{dt} = -\frac{n_l}{\tau_I} + S_I \\
\frac{dE}{dt} = \frac{-E}{\tau_E} + P_\alpha + P_{\text{ohmic}} - P_{\text{rad}} + P_{\text{aux}}
\]

In these equations, \(n_a\), \(n_n\), \(n_{D^3He}\), \(n_{DT}\), \(n_\alpha\) are the alpha particle, deuterium-tritium, deuterium-helium3 and the neutral fuel (defined as the number of fuel atoms divided by the core volume) and impurity densities, respectively. \(\tau_a\) is the confinement time for the alpha particles, \(S\) is the fueling rate, \(\tau_{DT}\), \(\tau_{D^3He}\) are the confinement time for ionized fuel particles of D, T and D, 3He, respectively. \(\tau_d\) is the controller lag time, \(E\) is the plasma energy, \(\tau_E\) is the energy confinement time, \(\tau_I\) is the confinement time for the impurities, \(S_I\) is the impurity injection rate, \(Q_\alpha = 3.52\text{MeV}\) is the energy of the alpha particles. \(P_\alpha\), \(P_{\text{aux}}\), \(P_{\text{rad}}\), \(P_{\text{ohmic}}\), \(P_i\) and \(P_{\text{fu}}\) are the alpha power, auxiliary power,
Ohmic power, radiation loss, the net plasma heating power and fusion power, respectively that are given for D–T and D–$^3$He fuels in the following:[5]

$$P_{\alpha DT} = \left( \frac{n_{DT}}{2} \right)^2 < \sigma v >_{DT} q_{\alpha DT} \quad (16-a)$$

$$P_{\alpha D^3He} = \left( \frac{n_{D^3He}}{2} \right)^2 < \sigma v >_{D^3He} q_{\alpha D^3He} \quad (16-b)$$

$$P_{auxDT} = \frac{E}{\tau_E} - \left( \frac{n_{DT}}{2} \right)^2 \langle \sigma v \rangle_{DT} q_{\alpha DT} - P_{ohmic} + A_{bDT} \left( n_{DT} + 2n_\alpha \right) \times$$

$$\left( n_{DT} + 4n_\alpha \right) \sqrt{\frac{2E_{DT}}{3N_{DT}}} \quad (17-a)$$

$$P_{auxD^3He} = \frac{E}{\tau_E} - \left( \frac{n_{D^3He}}{2} \right)^2 \langle \sigma v \rangle_{D^3He} q_{\alpha D^3He} - P_{ohmic} + A_{bD^3He} \left( n_{D^3He} + 2n_\alpha \right) \times$$

$$\left( n_{D^3He} + 4n_\alpha \right) \sqrt{\frac{2E_{D^3He}}{3N_{D^3He}}} \quad (17-b)$$

$E_i, N_i$ are the total energy and density for $(i = DT, D^3He)$:

$$E_{DT} = \frac{3}{2} N_{DT} T \quad (18-a)$$

$$E_{D^3He} = \frac{3}{2} N_{D^3He} T \quad (18-b)$$

and

$$N_{DT} = 2n_{DT} + 3n_{\alpha DT} + (Z_I + 1)n_I \quad (19-a)$$

$$N_{D^3He} = 2n_{D^3He} + 3n_{\alpha D^3He} + (Z_I + 1)n_I \quad (19-b)$$

$$P_{radDT} = A_{bDT} \times Z_{effDT} n_e^2 \sqrt{T} \quad (20-a)$$

$$P_{radD^3He} = A_{bD^3He} \times Z_{effD^3He} n_e^2 \sqrt{T} \quad (20-b)$$

where
\[ A_{bDT} = 4.85 \times 10^{-37} \left( \frac{Wm^3}{\sqrt{keV}} \right) \quad (21-a) \]
\[ A_{bD^3He} = 5.35 \times 10^{-37} \left( \frac{Wm^3}{\sqrt{keV}} \right) \quad (21-b) \]

The effective charge \((Z_{eff})\) and electron density \((n_e)\) for \(D-T\) and \(D-^3He\) are:

\[ Z_{effDT} = \frac{n_{DT} + 4n_{aDT}}{n_{DT} + 2n_{aDT}} \quad (22-a) \]
\[ Z_{effD^3He} = \frac{n_{D^3He} + 4n_{aD^3He}}{n_{D^3He} + 2n_{aD^3He}} \quad (22-b) \]
\[ n_{eDT} = n_{DT} + 2n_{aDT} + Z_I n_I \quad (23-a) \]
\[ n_{eD^3He} = n_{D^3He} + 2n_{aD^3He} + Z_I n_I \quad (23-b) \]

Where \(Z_I\) is the atomic number of impurities.

\[ P_{ohmicDT} = \eta_{DT} J^2 \quad (24-a) \]
\[ P_{ohmicD^3He} = \eta_{D^3He} J^2 \quad (24-b) \]

\(J\) is the plasma current density and \(n\) is electron density such that \(5 \times 10^5 \leq j \left( mA \right) \leq 1.5 \times 10^6\) and \(0 \leq n_e \left( m^{-3} \right) \leq 14 \times 10^{19}\). \(\eta\) is the Spitzer resistivity in which for \(D-T\) and \(D-^3He\) we have:[7,8]

\[ \eta_{DT} = 1.03 \times 10^{-4} \times T^{-\frac{3}{2}} \times Z_{effDT} \]
\[ \times \ln \left( \frac{T^{\frac{3}{2}}}{\sqrt{3.14 \times Z_{effDT} \times e^3 \times \sqrt{n}}} \right) \quad (25-a) \]
\[ \eta_{D^3He} = 1.03 \times 10^{-4} \times T^{-\frac{3}{2}} \times Z_{effD^3He} \]
\[ \times \ln \left( \frac{T^{\frac{3}{2}}}{\sqrt{3.14 \times Z_{effD^3He} \times e^3 \times \sqrt{n}}} \right) \quad (25-b) \]

Here \(T\) and \(e\) are the electron temperature and charge such that:

\[ e = 1.6 \times 10^{-19} \left( C \right) \]

\[ P_i = P_{\alpha_i} - P_{rad_i} + P_{aux_i} + P_{ohmic_i} \quad (26) \]

Where \(i=DT,D^3He\) and
\[ P_{fuDT} = \frac{(n_{DT})^2}{4} <\sigma v>_{DT} Q_{DT} \tag{27} \]

\[ P_{fuD^3He} = \frac{(n_{D^3He})^2}{4} <\sigma v>_{D^3He} Q_{D^3He} \tag{28} \]

Also, the gain (Q) of this type of reactor for the D-T and D-^3^He reactions D-T are given in the following:

\[ \tilde{Q}_{DT} = \frac{\tilde{P}_{fuDT}}{\tilde{P}_{auxDT}} \tag{29} \]

\[ \tilde{Q}_{D^3He} = \frac{\tilde{P}_{fuD^3He}}{\tilde{P}_{auxD^3He}} \tag{30} \]

Also from the plotting of equations (24-a) to (25-b) we have two-dimensional and three-dimensional graphs of resistivity and Ohmic power variations for ITER 90 HP fusion reactor in terms of electron density and temperature for D-T and D-^3^He in the temperature interval 0-100 keV. (see figures (3) and (4)).

Figure 3: Comparison of two-dimensional resistivity curves of two fuels a) D-T b) D-^3^He at three temperatures T = 10 keV, T = 70 keV, and T = 100 keV in terms of electron density variations.
Figure 4: Comparison of two-dimensional diagrams of the Ohmic power for two fuel a) D-T b) $D - ^3He$ at three different current density, $j = 0.5 \times 10^6 (A/m^2)$ and $j = 1 \times 10^6 (A/m^2)$ in terms of the electron density variations.

From viewing of two and three-dimensional graphs of resistivity and Ohmic power for both $D-T$ and $D-^3He$ fuel we find that both resistivity ($\eta$) and Ohmic power ($P_{\text{ohmic}}$) decrease by increasing of electron density at each temperature but increase by decreasing temperature. Energy confinement time of the reactor ITER90H-P is given:

$$\tau_E = f \cdot 0.82 \cdot I^{1.02} \cdot R^{1.6} \cdot B^{0.15} \cdot A_i^{-0.5} \cdot k_X^{-0.19} \cdot p^{-0.47} \quad (31)$$

Where the isotopic number for 50:50 D-T mixture percentage is 2.5. The factor scale $f$ depends on the confinement situation. $I \cdot R \cdot B$ are plasma current, plasma radius, toroidal plasma field and net plasma heating respectively, in which its numerical values are given in Table (4).

<table>
<thead>
<tr>
<th>$I$ (mA)</th>
<th>$R$ (m)</th>
<th>$P$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>6</td>
<td>75</td>
</tr>
</tbody>
</table>

Table (4): Numerical values of ITER90H-P fusion reactor[6]

Confinement times in terms of different values are scaled with energy confinement time $\tau_E$ as $\tau_d = k_d \cdot \tau_E$, $\tau_\alpha = k_\alpha \cdot \tau_E$, $\tau_{DT} = k_{DT} \cdot \tau_E$ and $\tau_{D^3He} = k_{D^3He} \cdot \tau_E$.

In addition, in our calculations we ignore of impurities because we interested to free conditions of impurities.

Results and Discussion

Note that, the values of variables in which obtain in the steady-state for fusion reactor ITER90H-P are shown with a line above them. So, the numerical values of these variables $\bar{n}_e \cdot n_\alpha \cdot n_{D,T} \cdot n_{D^3He}$ and energy state variable $E$ at refueling $S$ and auxiliary power $P_{aux}$ at steady state from solving of nonlinear point kinetic equations can be calculated with the inserting left side non-linear equations (6) to (15) equal to zero. Our obtained figures are given in the following for both $D-T$ and $D-^3He$ fuel at steady-state in temperature range 0 to 100 kev. It should be noted that the two parameters $K_d$ and $K_\alpha$ used in drawing the graphs are considered constant and its numerical values are $k_d = 3$ and $k_\alpha = 7$.
From Fig. 5 we see that particle density of D and T is declining because due to nuclear fusion of D – T, D and T particles are consumed and its amount are decreased. Then system go toward relative equilibrium and the value of densities could be fixed amount. Density of the alpha particles is increasing because due to D – T fusion reaction alpha particles are produced thus the value of them amount increases. Then since that they may have escaped from system thus the value of them are reduced and then since the system go toward to a relative equilibrium the numerical values of densities could be fixed amount. Neutral particle density of the fuel is injected at a constant rate $7 \times 10^{19} \text{ (m}^{-3})$ at all temperatures. Also, for the $D - ^3He$ reaction in temperature range of 0 to 100 kev, we see that alpha particle density, at first is increasing because as shown in Figure(2) to a temperature of 60 (keV), $\langle \sigma v \rangle_{D^3He}$ and therefore number of fusion reactions increase. As the temperature increases $\langle \sigma v \rangle_{D^3He}$ and thus number of fusion reactions decrease. Therefore alpha particle density is decreased. Initially the particle density of D and $^3He$ decrease because according to Figure (2) till temperature 70 (keV), $\langle \sigma v \rangle_{D^3He}$ and thus number of fusion reactions increase therefore more fuel is consumed. Therefore, the particle density of D and $^3He$ is reduced and these decreasing processes in this temperature range is perfectly obvious. Then in the temperature range 70 to 100 (keV), $\langle \sigma v \rangle_{D^3He}$ and thus number of fusion reactions of $D - ^3He$ are reduced. Therefore low $D - ^3He$ fuel is consumed, therefore gradually is increased. Neutral particle density of the fuel is injected at a constant rate at a tall temperature. Since that the magnitude of neutral density versus temperature is order of $6.98 \times 10^{17} \text{ (m}^{-3})$ therefore in comparison with the alpha particle, D, and T densities ($10^{19} \text{ (m}^{-3})$) is very low, that can be seen in the Figure (5). Also Figure (6) shows variations of energy density in terms of temperature for both fuel $D - T$ and $D - ^3He$ at steady state.
From Figure 6 for D−T and D−3He fuel we observe that initially the energy density of these fuel increases with increasing temperature until the temperature reaches the resonance and then gradually reduced. Its reason is that with increasing temperature number of fusion increases and thus more energy is produced such that in 70K eV which known as resonance temperature of D−T fusion reaction we have maximum energy. Then with increasing temperature, $(\sigma v)_{DT}$ is reduced. Therefore, the number of fusion reactions are reduced and thus energy is decreased. The same trend can be seen for the D−3He reaction except that since 50K eV is the resonance temperature of D−3He. Therefore, at the resonance temperature of D−3He fusion reaction we will have the maximum energy. With having the values of $\bar{n}_{DT}, \bar{n}_{n_{DT}}, \bar{n}_{n_{DT}}$, $\bar{n}_{D−He}, \bar{n}_{n_{D−He}}, \bar{n}_{n_{D−He}}$ and ignoring of impurities and using equations (17) to (20), (22) and (26) to (30) respectively, quantities of auxiliary power ($P_{aux}$), total energy density ($E$), total density ($\bar{N}$), radiative power loss ($P_{rad}$), effective charge of all ions ($Z_{eff}$), net power heating ($\bar{P}_i$), fusion power ($\bar{P}_f$), and fusion gain ($\bar{g}$) of fuel D−T and D−3He are calculated in terms of temperature variations and their curves are given in the Figure (7) to (15).

Figure 7: auxiliary power of a) D−T and b) D−3He at the steady state in the temperature range of 0 to 100 (keV).
According to Fig. 7 can be concluded that auxiliary power of D-T fuel initially is reduced and the temperature about $12(keV)$ is minimized then with increasing temperature is increased and at the $100(keV)$ temperature is maximized. For $D - ^3He$ this power is maximized at low temperatures then by increasing temperature auxiliary power is reduced and the temperature near $20(keV)$ is minimized and then by adding to temperature $100(keV)$ gradually grows.

Figure 8: Comparison of the total energy of the D-T and $D - ^3He$ fuel in the steady state and temperature range of 0 to 100 (keV).

According to the figure 8 we can find that the total energy of both D-T and $D - ^3He$ fuel with increasing temperature has increased linearly. Because according to equations (18 - a), and (18 - b) the total energy, directly proportional to temperature (T). Since from figure 9 we can see the total densities of $\bar{N}_{DT}$ and $\bar{N}_{D^{3}He}$ are nearly equal therefore according to the relations (18 - a), and (18 - b), we can conclude that the total energy $\bar{E}_{DT}$ and $\bar{E}_{D^{3}He}$ will be the same.

Figure 9: Comparison of total density of the D-T and $D - ^3He$ fuel in the steady state and temperature range of 0 to 100 (keV).
According to Figure 9 can be seen that the total density of the fuel D-T and $D^3He$ similar trends with temperature variations and with temperature increasing, their values rapidly increase and then reach to a constant value also from 80 (keV) to 100 kev grows gradually but at all temperatures, the density of $D^3He$ is greater than D-T. This behavior is due to the kinetic equations governing the system and the shape of equations (20-a) and (20-b).

![Figure 10: Comparison of the radiative loss power for both fuel of D-T and $D^3He$ in steady state at temperature interval 0 to 100 (keV)](image)

According to figure 10 can be found that radiative power loss of both fuel D-T and $D^3He$ with temperature increasing, grows. Because, as the temperature increases, the ions of the plasma, obtain more energy and thus these power increase. Also we can see that radiative power loss of $D^3He$ variations at all temperature range is greater than D-T.

![Figure 11: Comparison of effective charge of all ions for both D-T and $D^3He$ fuel in the steady state and temperature range of 0 to 100 (keV).](image)

According to figure 11 can be found that effective charge of all ions for D-T with increasing of temperature increase and its maximum value is about 1.99 at 5 kev then with increasing temperature its value is fixed. For $D^3He$ similar performance is occurred except that its maximum value at 24 kev is about 1.99 then with increasing temperature till 76 kev its value is
fixed and after that with growth of temperature its value is reduced very slowly. Also we can see that in the temperature interval 30 to 80 kev $Z_{\text{eff} DT}$ and $Z_{\text{eff} D^3He}$ are coincidence. Also from equation (26) net heating power for both fuel $D - T$ and $D - ^3He$ in the temperature interval 0 to 100 kev are plotted (see fig 12).

![Figure 12](image12.png)

**Figure 12:** Comparison of the net heating power of $D - T$ and $D - ^3He$ in steady state at temperature interval 0 to 100 (keV)

According to figure 12 can be found that the value of net heating power of the $D - T$ increases with increasing temperature such that its value changes from $4.12 \times 10^7 (W)$ to $4.31 \times 10^7 (W)$, and then its value slowly and gradually decreases with increasing temperature. But from this figure you can see that the $D - ^3He$ total net heating power is always greater than $D - T$, in the temperature range (0 to 100 keV).

![Figure 13](image13.png)

**Figure 13:** Comparison of fusion power for both of $D - T$ and $D - ^3He$ fuel in the steady state in terms of temperature variations.

According to figure 13 can be found that the fusion power of both fuels $D-T$ and $D - ^3He$ grows with temperature increasing. Since that the number of fusion reactions is enhanced therefore the fusion power increases. Also, we can see that the fusion power of $D-^3He$ fuel is higher than $D - T$.
According to Figure 14, it can be found that initially fusion gain of D-T fuel is enhanced with temperature increasing and its maximum value is about 6. Then, with temperature growing, its value gradually reduced such that at 100 keV temperature, its Q value reaches 5. However, D-\(^3\)He fusion gain with temperature increasing enhances such that at 20 keV temperature reaches to the value 0.012, than with increasing temperature up to 100 keV, its value is fixed. In general, from this figure we see that fusion gain of D-T is greater than D-\(^3\)He. So, in the steady state, it is not recommended to use the D-\(^3\)He fuel for the reactor ITER90H-P. For having more realistic information about Q for both fuel D-T and D-\(^3\)He, it is better to enter \(\tau_E\) changes on the fusion gain. (see Fig.15)

![Figure 14: Fusion gain variations of a) D-T, b) D-\(^3\)He fuel in the steady state at the temperature range 0 to 100 (keV) and \(\tau_{EDT} = 3.3(s)\), \(\tau_{ED\text{-He}} = 16(s)\)](image)

**Figure 15:** Three dimensional variation of fusion gain for D-T and D-\(^3\)He fuel, in the steady state at temperature range of 0 to 100 (keV) and \(\tau_E\) range of 0 to 30 seconds.

Also, if in determining of physical quantities such as radiative power loss, total energy, net heating power, auxiliary power, power, and fusion gain of both fuel D-T and D-\(^3\)He we enter the variations of \(K_d\), \(K_\alpha\) parameters in the range of 0\(\langle K_\alpha \leq 25,\)

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and also by considering fusion power variations for both fuel \( D - T \) and \( D - ^3He \) versus energy confinement time (\( \tau_E \)) the discussion is wider (see figs.16 to 20).

Figure 16: Three dimensional comparison of radiative power loss for D - T fuel in steady state at three temperature 10, 70 and 100 kev in terms of \( K_\alpha \) and \( K_d \) variations.

With seeing Fig.16 we find that the radiation power loss of D-T fuel increased with increasing temperature and \( K_\alpha \) and \( K_d \).

Figure 17: Three dimensional comparison of total energy for D - T fuel in steady state at three temperature 10, 70 and 100 kev in terms of \( K_\alpha \) and \( K_d \) variations.

From Figure17 can be seen that the total energy of the \( D - T \) fuel increases with increasing temperature as \( K_\alpha \) and \( K_d \) change.
Figure 18: Three dimensional comparison net heating power for D – T fuel in steady state at three temperature 10, 70 and 100 kev in terms of $K_\alpha$ and $K_d$ variations. From Figure 18 can be seen that the net heating power decreases with increasing temperature as $K_\alpha$ and $K_d$ change.

Figure 19: Three dimensional comparison auxiliary power for D – T fuel in steady state at three temperature 10, 70 and 100 kev in terms of $K_\alpha$ and $K_d$ variations. We have seen from Fig.19, by increasing temperature auxiliary power ($P_{aux}$) will rise as $K_\alpha$ and $K_d$ change.
Figure 20: Three dimensional variation of fusion gain for D - T and \( D - ^3He \) fuel in the steady-state at temperature range of 0 to 100 (keV) and energy confinement time range 0 to 30 seconds.

According to figure 20 we can say that in all temperature and confinement time variations the fusion power \( D - ^3He \) is greater than D-T.

**Conclusions**

With studying and analyzing of ITER90H-p fusion reactor sand solving the non-linear point kinetic equations governing on the two-fuel D - T and \( D - ^3He \) at steady, dynamical and perturbation states, we find that the main quantities in determining the fusion gain are the densities of alpha particles, deuterium, tritium, helium, neutral fuel, electron, fusion energy and the total energy, and density of total particles, the effective charge of all ions, radiative powerless, auxiliary and fusion power, respectively. In order to be commercially competitive, a fusion reactor needs to run long periods of time in a stable burning plasma mode at working points which are characterized by a high Q, where Q is the ratio of fusion power to auxiliary power. Active burn control is often required to maintain these near-ignited or ignited conditions ( \( Q = \infty \) ). Although operating points with these characteristics that are inherently stable exist for most confinement scaling, they are found in a region of high temperature and low density. Our studies show that in the steady-state above quantities are only a function of temperature and each has its own specific variations and also at the temperature 70 (keV) these quantities produce the maximum fusion gain for both of fuels D - T and \( D - ^3He \), such that their values are equal to 6.01 for D - T fuel and the 0.012 for \( D - ^3He \), respectively. Fusion using \( D - ^3He \) fuel requires significant physics development particularly of plasma confinement in high performance alternate fusion concepts. Countering that cost, engineering development cost should be much less for \( D - ^3He \) than D - T, because \( D - ^3He \) greatly ameliorates the daunting obstacles caused by abundant neutrons and the necessity of tritium breeding. A \( D - ^3He \) fusion fueled fusion reactor would also possess substantial safety and environmental advantages over D - T. Currently recommended for reactor ITER90H-p is used D - T fuel and still need more research to be done on the fuel \( D - ^3He \).
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APPENDIX

To obtain thenumerical values of the density state $n_I n^\alpha D^+T$ variables we solve analytically point kinetic nonlinear equations governing on the two fusion fuels $D – T$ and $D – ^3He$. Because, the equations of the two fuel sare similar therefore we solve only the equations (6) to (9) for $D – T$ fuel. We put theleftsides of equations (6) to (9) equal to zero

$$\frac{dn_I}{dt} = \frac{dn_I}{dt} = \frac{dn_a}{dt} = \frac{dn_n}{dt} = 0$$

So, from putting the equation (1) equal to zero, we have:

$$-\frac{n_{DT}}{\tau_{DT}} - 2\langle n_{DT} \rangle^2 <\sigma\nu>_{DT} + \frac{n_n}{\tau_d} = 0 \quad (A-1)$$

$$\langle n_{DT} \rangle^2 <\sigma\nu>_{DT} = \frac{n_a}{\tau_\alpha} \quad (A-2)$$

According to equation(A-1)and(A-2), $n_{DT}$ isas follows:

$$(A-3) \quad n_{DT} = \left[ \frac{4n_a}{<\sigma\nu>_{DT} \tau_\alpha} \right]^{1/2} \times$$
\[
\begin{align*}
&-1\left[\frac{4n_a}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} - \\
&2\left(\frac{1}{2}\right)^{2}\left[\frac{4n_a}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} - 1\left[\frac{4n_a}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} - 2 \frac{n_a}{\tau_a} + \frac{n_a}{\tau_d} = 0 \\
&<\sigma'\rangle_{DT} + \frac{n_a}{\tau_d} = 0 \\
&\text{(A-4)} \quad \bar{n}_n = \left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} + 2 \frac{n_a}{\tau_a} \tau_d
\end{align*}
\]

We put equation (8) equal to zero (A-5) \( -\frac{n_n}{\tau_d} + s = 0 \)

By inserting equation (A-4) inside to (A-5) the following relation is given:

\[
\begin{align*}
&-1\left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} + 2 \frac{n_a}{\tau_a} \tau_d + s = 0 \\
&\text{(A-6)} \quad 2 \frac{n_a}{\tau_a} + \frac{2}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} \left[\frac{n_a}{\tau_a}\right]\frac{1}{2} = s
\end{align*}
\]

We define \( x \) as follows and insert it into the equation (A-6):

\[
\begin{align*}
&(A-7) \quad x = \left[\frac{n_a}{\tau_a}\right]\frac{1}{2} \\
&2x^2 + \frac{2}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} x = s \\
&2x^2 + \frac{2}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} x - s = 0
\end{align*}
\]

The above equation is a quadratic equation, by solving this equation \( x \) is determined and by replacing into eq (A-4) we have

\[
\bar{n}_n = \left[\frac{2}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} + 2 \frac{n_a}{\tau_a} \right] \tau_d
\]

And using equation (A-7) \( \bar{n}_n \) can be written as:

\[
\text{(A-8)} \quad \bar{n}_n = \left[\frac{2}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2} x^2 + x \right] \tau_d
\]

Here the variable defined as follows and insert it inequation (A-9)

\[
y = \frac{2}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\left[\frac{1}{\tau_{DT}\langle\sigma'\rangle_{DT} \tau_a}\right]\frac{1}{2}
\]
we put the equation (7) equal to zero:

\[
\frac{-n_{DT}}{\tau_{DT}} - 2\left(\frac{n_{DT}}{2}\right)^2 <\sigma v >_{DT} + \frac{n_n}{\tau_d} = 0
\]

Then by inserting equation (A-10) into equation (A-11) we get the following quadratic equation in terms of \(n_{DT}\):

\[
2 \quad \frac{n_{DT}}{2} \quad <\sigma v >_{DT} + \frac{n_{DT}}{\tau_{DT}} - \left[ y \frac{1}{2} + 2x \right] = 0
\]

by solving this equation we have

\[
\frac{1}{2} \left[ \frac{1}{(\tau_{DT})^2} + 2 <\sigma v >_{DT} \left[ y \frac{1}{2} + 2x \right] \right]^{\frac{1}{2}}
\]

Electron density in the steady state is obtained from the relationship:

\[
\frac{1}{4} \left( \frac{-1}{2\tau_{DT}} + \frac{1}{2} \left[ \frac{1}{(\tau_{DT})^2} + 2 <\sigma v >_{DT} \left[ y \frac{1}{2} + 2x \right] \right] \right)^{\frac{1}{2}} \times <\sigma v >_{DT} = \frac{n_a}{\tau_a}
\]

If we ignore the effect of impurities the electron density equation is as follows:

\[
\frac{1}{4} \left( \frac{-1}{2\tau_{DT}} + \frac{1}{2} \left[ \frac{1}{(\tau_{DT})^2} + 2 <\sigma v >_{DT} \left[ y \frac{1}{2} + 2x \right] \right] \right)^{\frac{1}{2}} \times <\sigma v >_{DT} = \overline{n}_{eDT} = \overline{n}_{DT} + 2\overline{n}_{aDT} + Z_i \overline{n}_1.
\]

By substituting equations (A-12) and (A-13) in equation (A-14), the steady-state electron density is given by (A-15):

\[
\frac{1}{4} \left( \frac{-1}{2\tau_{DT}} + \frac{1}{2} \left[ \frac{1}{(\tau_{DT})^2} + 2 <\sigma v >_{DT} \left[ y \frac{1}{2} + 2x \right] \right] \right)^{\frac{1}{2}} \times <\sigma v >_{DT} = \overline{n}_{eDT} = \overline{n}_{DT} + 2\overline{n}_{aDT}
\]

It should be noted that the above relationships are true for \(D - ^3He\) fuel except that in the mentioned relations the index \(DT\) must be replaced by \(D ^3He\).