Reciprocal Symmetric Kinematics and Correspondence between Special Relativity and Quantum Mechanics

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Abstract

We have defined reciprocal symmetry and observed its presence in Special Relativity and quantum mechanics. Therefore, reciprocal symmetry relates Special Relativity to quantum mechanics. Reciprocal symmetry implies that there is a boundary value velocity $c$, which is the limiting value and which is also constant under addition. We have, then, developed reciprocal symmetric law of addition of velocities. In case of collinear velocities it agrees with Lorentz-Einstein law of addition. General reciprocal symmetric transformation is different from General Lorentz transformation. Reciprocal symmetric transformation is associative and complex. A comparison between RST and LT is given.

Keywords: Reciprocal Symmetry; Reciprocal Symmetric Transformation; Lorentz Transformation; Associativity

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INTRODUCTION

Planck and Einstein: In 1900 Planck’s hypothesis set a lower limit to energy quantum. In 1905 Einstein suggested velocity has an upper limit. If a quantity has an upper limit, its reciprocal has a lower limit. This suggests a reciprocal relation between Planck’s hypothesis and Einstein’s postulate.

Slowness: Corresponding to every velocity $v = x/t$, we can define slowness $v' = 1/v = t/x$. Physics is independent of the way we describe it. The principle of objectivity demands that a description in terms of velocity should be as valid as the description in terms of slowness. We intend to study how this velocity-slowness symmetry relates Special Relativity to quantum mechanics.

Reciprocal Number: Every number (except 0) has a reciprocal. If we can establish isomorphism between numbers and their reciprocals, the description of an event in terms of reciprocals of numbers should be as valid as a description in terms of numbers themselves.


**Reciprocity between Relativity and Quantum Mechanics**

From the velocity \( v \) we form the dimensionless quantity \( v/c \) where \( c \) is an arbitrarily chosen (scale) velocity. We define the reciprocal velocity, \( u^* \), and slowness \( u' \) corresponding to velocity \( u \) by

\[
\frac{u^*}{c} = \frac{u'}{c'} = \frac{c}{u}
\]

(1)

**Reciprocity between Relativity and De Broglie Relation**

Consider Einstein’s relation

\[
mc^2 = E = hv
\]

(2)

And the wavelength for a particle with velocity \( u \) is given by de Broglie relation

\[
\lambda = \frac{h}{m.u}
\]

(3)

Combining (2) and (3), we get the velocity, \( u^* \) for de Broglie wave

\[
u^* = \lambda v = \frac{hv}{m.u} = \frac{c^2}{u}
\]

(4)

**Reciprocity between Einstein’s Postulate and Planck’s Hypothesis**

Let \( E_u \) be 2 times the kinetic energy of a particle with velocity \( u \)

\[
E_u = mu^2
\]

(5)

According to Einstein’s postulate

\[
|u| \leq c
\]

(6)

Therefore, using (5) and (6) and replacing \( u \) by \( c \)

\[
E_u \leq E_c
\]

(7)

Corresponding to (2) we define reciprocal energy

\[
E_u^* = \frac{(E_c)^2}{E_u} = \frac{(mc^2)^2}{m.u^2} = m\left(\frac{c^2}{u}\right)^2 = m(u^*)^2 = E_u^*
\]

(8)

Therefore, corresponding to (7) we have

\[
E_u^* \geq E_c
\]

(9)
We relate

\[ E_c = h \nu \]  

(10)

Where \( m \) will determine \( \nu \). By (9) \( E_c \) sets the lower limit of \( E_u^* \) just as by (7) it sets the upper limit of \( E_u \). Therefore, we may state Einstein’s postulate (and Planck’s hypothesis) in two equivalent ways as below:

**Einstein’s picture:** When a body gains (or loses) energy \( E_u \) the gain (or loss) cannot be greater than \( E_c \).

**Planck’s picture:** When a body gains (or loses) energy \( E_u^* \) the gain (or loss) cannot be less than \( E_c \).

In terms of speed the above statement will be:

**Einstein’s picture:** When a body gains (or loses) speed \( u \) the gain (or loss) of speed cannot be greater than \( c \).

**Planck’s picture:** When a body gains (or loses) reciprocal speed \( u^* \) the gain (or loss) of reciprocal speed cannot be less than \( c \).

**Discreteness**

Consider two reciprocal energy levels \( E_u^* \) and \( E_v^* \). Let the difference between the reciprocal energy levels be \( E_{u-v}^* \). Since \( E_{u-v}^* \) is a reciprocal energy (group property), it fulfills condition (9)

\[ E_{u-v}^* \geq E_c \]  

(11)

The above relation shows that the difference between any two reciprocal energy levels cannot be less than \( E_c \) which is a non-zero quantity. Therefore, reciprocal energy is discrete. Similarly, let the difference between two reciprocal speed levels \( u^* \) and \( v^* \) be \( d_{u-v}^* \). By the requirement of group property \( d_{u-v}^* \) is also a reciprocal speed. Therefore, by (1)

\[ |d_{u-v}^*| \geq c \]  

(12)

Therefore, the minimum difference between two reciprocal speed levels is a non-zero reciprocal speed. Therefore, reciprocal speeds are discrete quantities.
Reciprocal Symmetry

To reflect the symmetry mentioned in sections 1 and 2, we postulate that the law of addition (composition) of velocities ⊕ should fulfill the symmetry condition

*Postulate:*

\[ u \oplus v = (u^*) \oplus (v^*) \]  \hspace{1cm} (13)

(13) implies\(^1\)

\[ u \oplus (v^*) = (u^*) \oplus v = (u \oplus v)^* \]  \hspace{1cm} (14)

(14) may also be written as

\[ \{u \oplus (v^*)\} \equiv (u \oplus v) = c^2 \]  \hspace{1cm} (15)

If \( v = c \) (1) shows \( v^* = c \) and (15) gives the invariance\(^2\) under addition

\[ c \oplus v = \pm c \]  \hspace{1cm} (16)

(16) should be valid for all values of \( v \) including 0. Therefore,

\[ c \oplus v = c \]  \hspace{1cm} (17)

Motion in One Dimension

Two definitions of \( \oplus (\oplus \text{ and } \oplus^*) \) fulfill requirement (13)

\[ u \oplus v = (u^*) \oplus (v^*) = \frac{u + v}{1 + u.v/c^2} \]  \hspace{1cm} (18)

And

\[ u \oplus^* v = (u^*) \oplus^*(v^*) = \frac{uv + c^2}{u + v} \]  \hspace{1cm} (19)

We choose \( \oplus \) and \( \oplus^* \) such that

\[ |u \oplus v| \leq c \text{ if } |u| \leq c \text{ and } |v| \leq c \]  \hspace{1cm} (20)

And

\[ |u \oplus^* v| \geq c \text{ if } |u| \geq c \text{ and } |v| \geq c \]  \hspace{1cm} (21)

(18) is Lorentz-Einstein law of addition of velocities and (19) gives the law of addition of de Broglie velocities. Both of them fulfill requirement (13)
GENERAL LORENTZ TRANSFORMATION AND RECIPROCITY

The general Lorentz transformation is given by

\[ w = u \oplus E (-v) = \frac{u - v}{1 - u \cdot v / c^2} - \frac{1}{c^2} \gamma_u \frac{u \times (u \times v)}{\gamma_u + 1 - u \cdot v / c^2} \] (22)

Where

\[ \gamma_u = \frac{1}{\sqrt{1 - (u / c)^2}} \] (23)

Its properties under reciprocal inversion corresponding to (15) are

\[ \{ u \oplus E v^* \} \cdot \{ u \oplus E v \} = c^2 \text{ for } v^* \cdot v = c^2 \] (24)

But

\[ \{ u^* \oplus E v \} \cdot \{ u \oplus E v \} \neq c^2 \text{ for } u^* \cdot u = c^2 \] (25)

(22) does not have a symmetry property comparable to (13). (23) and (23) show that Lorentz transformation is partially reciprocal symmetric, but it reveals a pathology that Lorentz transformation does not treat the two velocities equally. This is against the principle of relativity.

RECIPROCAL SYMMETRIC TRANSFORMATION

We want to replace \( u \oplus E v \) of (22) by \( u \oplus RS v \) (\( E \) is replaced by \( RS \)), which should have property (13). Reciprocal symmetry (13) will ensure (16).

\[ w_{RS} = u \oplus RS (-v) = \frac{u - v - iu \times v / c}{1 - u \cdot v / c^2} \] (26)

We shall call \( u^* \) a reciprocal of \( u \) if

\[ u \cdot u^* = c^2 \text{ and } (u^*)^* = u \] (27)

For arbitrary \( G \)

\[ u^* = \frac{G + iG \times u / c}{u \cdot G / c^2} \text{ and } (u^*)^* = u \] (28)

And corresponding to (13) we have the symmetry relation

\[ w_{RS} = u^* \oplus RS (-v^*) = u \oplus RS (-v) \] (29)
The reciprocal sum corresponding to (19) and for arbitrary $G$ will be

$$w_{RS} = u \oplus^*_RS v = \frac{G.c^2 + (u \cdot v)G - G \times (u \times v) + i.c.G \times (u + v)}{G \cdot (u + v) + i.G \cdot (u \times v)/c}$$

(30)

Or, if $G = u \oplus RS v$

$$w_{RS} = u \oplus^*_RS v = \frac{c^2 + u \cdot v}{u^2 + v^2 + 2.u \cdot v - (u \times v/c)^2} \{u + v + iu \times v/c\}$$

(31)

**HYPERBOLIC TRANSFORMATION AND RELATIVISTIC VELOCITY**

Galilean velocity $U$ corresponding to relativistic velocity $u$ is defined by

$$U/c = \text{arctanh}(u/c) \quad \text{or} \quad u/c = \tanh(U/c)$$

(32)

c is a constant speed. (32) permits us to write the following identity

$$U/c = \ln \frac{1 + u/c}{\sqrt{1 - (u/c)^2}} \quad \text{with} \quad -1 \leq u/c \leq 1$$

(33)

The Galilean relative velocity $W = U - V$ becomes, using (33)

$$W = U - V = c ln \frac{1 + u/c}{\sqrt{1 - (u/c)^2}} \frac{1 - v/c}{\sqrt{1 - (v/c)^2}} = c \ln \frac{1 + w/c}{\sqrt{1 - (w/c)^2}}$$

(34)

Where the corresponding relativistic sum is

$$w = u \oplus (-v) = \frac{u - v}{1 - u.v/c^2}$$

(35)

**LORENTZ INVARIANCE AND RECIPROCITY**

We can rewrite (33) including a $t$ as below

$$Ut/ct = \ln \frac{1 + ut/ct}{\sqrt{1 - (ut/ct)^2}}$$

(36)

Replacing $Ut \rightarrow X$ and $ut \rightarrow x$ in (33) we can write

$$X/ct = \ln \frac{ct + x}{\sqrt{(ct)^2 - (x)^2}}$$

(37)
Corresponding to (34) we can write

\[
X'/c_0 = X/c_0 - Vt/c_0 = \ln \frac{ct + x}{\sqrt{(ct)^2 - x^2}} - \frac{1 - v_0t}{\sqrt{1 - (v_0/c)^2}} = \ln \frac{ct' + x'}{\sqrt{(ct')^2 - x'^2}}
\]  

(38)

Where

\[
t' = \gamma_v (t - x_0v/c^2) \quad \text{and} \quad x' = \gamma_v (x - vt)
\]  

(39)

Using (39)

\[
(ct')^2 - (x')^2 = (ct)^2 - (x)^2
\]  

(40)

Using (38), (39) and (40)

\[
(X' - X')/c_0 = (V - V)/c_0 = \ln \frac{1 - (v_0/c)^2}{1 - (v/c)^2} = 2. \ln \frac{\sqrt{1 - (v/c)^2}}{\sqrt{1 - (v_0/c)^2}} = 0
\]  

(41)

Therefore,

\[
(X' - X')/c_0 + \ln(t_0^2) = \ln \frac{t_0}{\sqrt{1 - (v_0/c)^2}} + t_0 \sqrt{1 - (v/c)^2} = \ln(t_T t_L) = \ln(t_0^2)
\]  

(42)

Where

\[
t_T = \frac{t_0}{\sqrt{1 - (v_0/c)^2}} \quad \text{and} \quad t_L = t_0 \sqrt{1 - (v/c)^2}
\]  

(43)

Lorentz invariance (40) and definitions (43) do not contain any more information than the trivial relations

\[
(X' - X')/c_0 = (V - V)/c_0 = 0 \quad \text{or} \quad \ln(t_0^2) = \ln(t_0 t_0)
\]  

(44)

CONCLUSION

We have developed much of Special Relativity without invoking Einstein’s postulate. Reciprocal symmetry also relates Special Relativity to quantum mechanics.

REFERENCES

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